MATH3280A Introductory Probability, 2014-2015
Solutions to HW2

P.105 Ex.1

Solution
Let R be the event that the signal received is a dot.
Let T be the event that the signal is a transmitted dot.
By Bayes' Formula,

$$P(T|R) = \frac{P(R|T)P(T)}{P(R|T)P(T) + P(R|T^c)P(T^c)}$$

$$= \frac{\frac{3}{4} \times 0.4}{\frac{3}{4} \times 0.4 + \frac{1}{3} \times 0.6}$$

$$= \frac{3}{5}.$$

The probability that a dot received was actually a transmitted dot is $\frac{3}{5}$.

$\square$

P.106 Ex.5

Solution
Let E be the event that a certain employee makes over \$ 120,000 a year.
Let W be the event that a certain employee is a woman.
By Bayes' Formula,

$$P(W|E) = \frac{P(E|W)P(W)}{P(E|W)P(W) + P(E|W^c)P(W^c)}$$

$$= \frac{0.02 \times 0.3}{0.02 \times 0.3 + 0.05 \times 0.7}$$

$$= \frac{6}{41} \approx 0.1463$$

About 14.63% of employees who make over \$ 120,000 a year are women.

$\square$

P.119 Ex.4

Solution
The sample space is $\Omega = \{1, 2, ..., 6\} \times \{1, 2, ..., 6\}$.
We have

$$\begin{aligned} A &= \{(m, n) \in \Omega : m + n \ is \ odd\} \\ &= \{(1, 2), (1, 4), (1, 6), (2, 1), (2, 3), (2, 5), ..., (6, 5)\} \end{aligned}$$

$$\begin{aligned} B &= \{(m, n) \in \Omega : m = 2\} \\ &= \{(2, 1), (2, 2), ..., (2, 6)\} \end{aligned}$$

$$AB = \{(2, 1), (2, 3), (2, 5)\}$$

Then

$$\begin{aligned} P(A) &= \frac{|A|}{|\Omega|} = \frac{18}{36} = \frac{1}{2} \\ P(B) &= \frac{|B|}{|\Omega|} = \frac{6}{36} = \frac{1}{6} \\ P(AB) &= \frac{|AB|}{|\Omega|} = \frac{3}{36} = \frac{1}{12} \end{aligned}$$

Since $P(AB) = P(A)P(B)$, A and B are independent.

$\square$